

LECTURE NO 9

Determine the divergence of these vector fields:

(a) $\mathbf{P} = x^2yz \mathbf{a}_x + xz \mathbf{a}_z$

(b) $\mathbf{Q} = \rho \sin \phi \mathbf{a}_\rho + \rho^2 z \mathbf{a}_\phi + z \cos \phi \mathbf{a}_z$

(c) $\mathbf{T} = \frac{1}{r^2} \cos \theta \mathbf{a}_r + r \sin \theta \cos \phi \mathbf{a}_\theta + \cos \theta \mathbf{a}_\phi$

$$\begin{aligned} \text{(a) } \nabla \cdot \mathbf{P} &= \frac{\partial}{\partial x} P_x + \frac{\partial}{\partial y} P_y + \frac{\partial}{\partial z} P_z \\ &= \frac{\partial}{\partial x} (x^2 yz) + \frac{\partial}{\partial y} (0) + \frac{\partial}{\partial z} (xz) \\ &= 2xyz + x \end{aligned}$$

$$\begin{aligned} \text{(b) } \nabla \cdot \mathbf{Q} &= \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho Q_\rho) + \frac{1}{\rho} \frac{\partial}{\partial \phi} Q_\phi + \frac{\partial}{\partial z} Q_z \\ &= \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho^2 \sin \phi) + \frac{1}{\rho} \frac{\partial}{\partial \phi} (\rho^2 z) + \frac{\partial}{\partial z} (z \cos \phi) \\ &= 2 \sin \phi + \cos \phi \end{aligned}$$

$$\begin{aligned} \text{(c) } \nabla \cdot \mathbf{T} &= \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 T_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (T_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} (T_\phi) \\ &= \frac{1}{r^2} \frac{\partial}{\partial r} (\cos \theta) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (r \sin^2 \theta \cos \phi) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} (\cos \theta) \\ &= 0 + \frac{1}{r \sin \theta} 2r \sin \theta \cos \theta \cos \phi + 0 \\ &= 2 \cos \theta \cos \phi \end{aligned}$$